

A Numerical Comparative Study on Heat Generation Pattern for Different Materials Based on Eigenfrequencies

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Abstract- : Eigenfrequencies also known as natural frequencies, are certain discrete frequencies at which a system tends to vibrate. Natural frequencies are inherent, hence all bodies, whether they are made by man or by nature have their own set of natural frequencies that they tend to vibrate at. For instance, man-made musical instruments like the harp, have their own set of natural frequencies, and when they vibrate at these frequencies, they give rise to standing waves. In mechanical structures such as building or bridges, eigenfrequencies are of special interest and concern, as they can be used to predict when a structure will fail. In this paper, we will analyse eight fundamental eigenfrequency values for five different materials. Each material studied is used to manufacture spur gears for various applications. There is also a trend in variation of temperature of the material with its respective eigenfrequencies. The purpose of this paper is to investigate the variation of temperature with eigenfrequencies for different materials studied as well as report on the most suitable material based on the minimum vibration observed.

Keywords: Eigenfrequencies, spur gear, natural frequencies.

1. INTRODUCTION

When vibrating at a certain frequency, it has been observed that a material deforms into a distinct shape. This specific frequency is called the eigenfrequency or the mode. In structural analysis, this is of extreme importance, because the knowledge and understanding of eigenfrequencies can give engineers precise understanding regarding when a structure is going to fail. Throughout the years, significant work has been done on eigenfrequencies and how they affect certain structures. For instance, Gudmundson et al. investigated a method which detects variations in eigenfrequencies that result due to changes in a structure, due to cracks or notches [2]. Witczak et al. [3] described a calculation method for estimating natural frequencies for lower order modes. The work was done to analyze the rotation caused by forces acting on the stator of an AC (alternating current) machine. Chaari et al. [4] investigated the dynamic response of a planetary gear set by applying an iterative method, which relied on extracting the eigenfrequencies of the gears.

It is now known that there is a link between eigenfrequency or mode and both deformation and heat generation within a material. In a study by Tariqul et al. [5], it has been shown using simulation methods that stress causes formation of different heat patterns in different materials. The reason behind the formation of

heat is governed by the change in internal energy showed in first law of thermodynamics [6]. After analyzing the consequences of eigenfrequency on the structure and the resultant stress formation, it can be deduced that there is a relation between the natural frequency and heat formation in the corresponding structure.

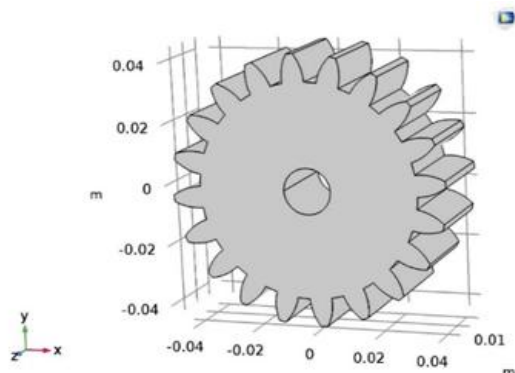
The uniqueness of this paper is the fact that it focusses on extracting the first eight fundamental frequencies of five different materials using COMSOL Multiphysics software, primarily for the purpose of investigating the deformation of each material at each specific eigenfrequency. Previous work has focused on extracting eigenfrequencies primarily for design purposes. For this analysis, we have selected spur gear model for our various tests. The reason behind the selecting gear for this analysis is that it works under both rotational motion with a certain vibration as well as under dynamic load. In our study, we will focus on how much the deformation of a spur gear changes with each eigenfrequency or mode, hence that will provide useful information regarding the deformation pattern in a body with each corresponding eigenfrequency

2 FINITE ELEMENT MODELING OF THE SPUR

2.1 Geometry

For the purpose of analysis, first the CAD model of the

gear was designed using the SOLIDWORKS premium 2019 software. The geometry of the gear holds the following specifications as given in the Figure 1(b). After designing the model, it was converted into compatible file format and imported in COMSOL Multiphysics.



Parameters	Specifications
Module	4
Number of teeth	20
Pressure angle	20 degree
Face width	30 mm
Hub style	One side
Hub diameter	25 mm

FIGURE 1. (a) Geometry of the spur gear (b) Parameters of the gear

2.1.1 Material Selection

For this study, five different materials were selected, namely AISI 4340 steel, Aluminum, Copper, Nylon and Titanium each with the same parameters as displayed in the table below. All these materials are more or less used for constructing gears used for different purposes.

TABLE 1. Mechanical properties of the gear materials

Name of Material	Young's modulus (Pa)	Poisson's ratio	Density (Kg/m ³)	Thermal conductivity (W/mK)	Coefficient of thermal expansion (1/K)
AISI 4340 steel	205e9	0.28	7850	44.5	12.3e-6
Aluminum	69e9	0.33	2730	155	23.2e-6
Copper	110e9	0.35	8960	400	17e-6
Nylon	2e9	0.4	1150	0.26	280e-6
Titanium	105e9	0.33	4940	7.5	7.06e-6

2.1.2 Meshing and Boundary Conditions

The meshing operation was conducted using COMSOL Multiphysics meshing module. For the meshing of the model, physics-controlled meshing was used and for grid

generation, extremely fine topology was selected. The total number of generated elements were 322356 and the type of elements were tetrahedral. For defining boundary condition, the hub of the gear was made fixed to the shaft to be rotated and an rpm of 4000 was applied on the system. In the study option, the frequency corresponding to the RPM was chosen around which the eigenfrequencies are to be determined.

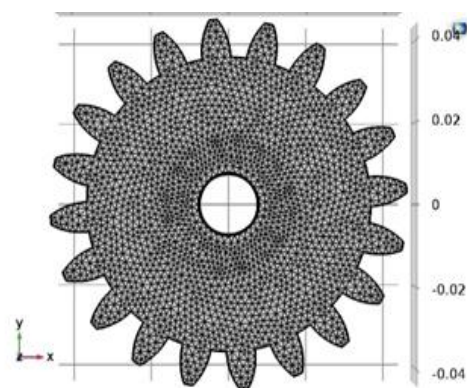


FIGURE 2 Meshing of the model

For the analysis, the following assumptions were made: i) No frictional loss between the driver and driven gears. ii) Steady operating conditions exist. iii) The entire system is isentropic. iv) No gaps between meshing teeth. v) Uniform applied torque on driver gear.

2.1.3 Governing Equations

The physics model selected for this study was Thermoelasticity, due to the fact that this particular model enables the determination of heat generation as well as deformation from each eigenfrequency calculated. The governing equations for this model are as follows. Mathematically, the model is represented as:

$$-\rho\omega^2 u - \nabla \cdot \sigma = F \quad (1)$$

$$-i\omega = \lambda \quad (2)$$

$$i\omega\rho c_p T - \nabla \cdot (k \nabla T) + Q - T_0 \alpha \nabla \cdot \sigma = 0 \quad (3)$$

$$i\omega\sigma - i\omega = \lambda \quad (4)$$

Where ρ and ρ_0 = density and original density respectively,

ω = frequency
 u = velocity vector
 ∇ = del operator
 σ = principal stress
 F = Force vector
 λ = Lamé Constant

i = imaginary number, c_p = coefficient of pressure
 T and T_0 = temperature and original temperature respectively
 Q = Heat generated, ε = strain

Under the thermoelastic material tab, the following equations define the properties of the materials:

$$s - S_0 = C: (\varepsilon - \varepsilon_0 - \varepsilon_{inel}) \dots \dots (5)$$

$$\varepsilon = \frac{1}{2} [(\nabla u)^T + \nabla u] \dots \dots \dots (6)$$

Here, s and S_0 = entropy and initial entropy respectively,
 ε , ε_0 and ε_{inel} = strain, initial strain and inelastic strain respectively.

Furthermore, equations (1) and (2) represents the equation for power, (3) represents the equation of the first law of thermodynamics, (5) represents the equation for the entropy of the system and (6) represents the equation of strain

3. Results and discussion

The link between eigenfrequency and deformation has been established in this study. From the graph below, we can see that each mode number corresponds to a different eigenfrequency value. Aluminium and AISI 4340 constantly have the highest values of eigenfrequencies. The higher the eigenfrequency, the better the material will be able to withstand damage due to vibrations. From the data above, we clearly see that AISI 4340 steel and aluminium have the highest eigenfrequencies at each mode. This indicates that these two structures, AISI 4340 in particular, are more capable of withstanding damage due to large vibrations and hence are more suitable to build structure and mechanical tools such as gears. Nylon on the other hand, displays very low eigenfrequencies, proving that even though it is lighter than steel and are capable of withstanding high bending stresses [7], under large vibrations they are highly inapt, and will resonate at very low frequencies in contrast to metallic gears.

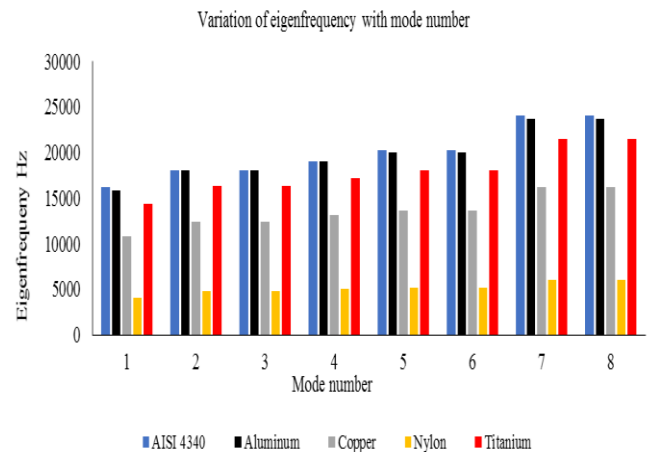


FIGURE 3. (a) Graph of eigenfrequency against material

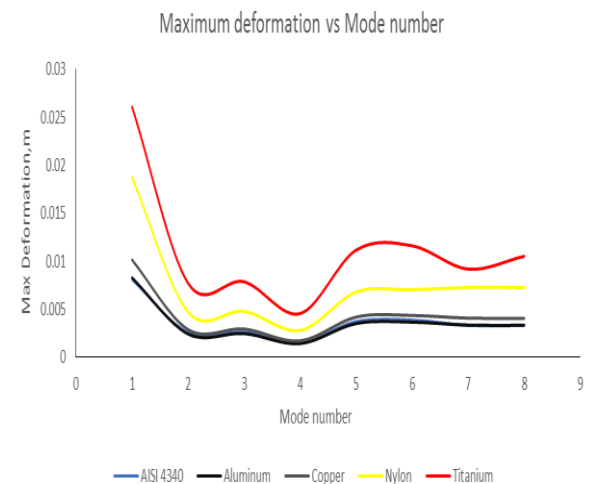


FIGURE 3. (b) Graph of deformation against mode number

The graph above shows the variation of maximum deformation against mode number. Titanium is seen to have very high deformation, proving it is unsuitable for use in high strength application gears. Conversely, AISI 4340 is seen to have the least deformation, this confirms that AISI is the best material for use in high strength gears and this is the primary reason why they are used in gears and gear applications.

4. CONCLUSION

To conclude and summarize, this paper focusses on two key findings and observations. The first being how eigenfrequencies or natural frequencies have an effect on maximum deformation and thus are responsible for damages within a structure. This knowledge allows engineers to determine the mode associated with the maximum deformation attained. Secondly, the

magnitude of eigenfrequencies associated with each mode indicates the suitability of the material for practical applications. Materials having higher eigenfrequencies are more stable and less prone to deformation and damage. Therefore, with keeping in mind various other factors such as cost and availability, materials having higher eigenfrequencies for a give mode are more desired than lighter and cheaper materials having lower eigenfrequencies over the same mode.

5. REFERENCES

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